Automated and Accurate Geometry Extraction and Shape Optimisation of 3D Topology Optimisation Results

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Abstract

Topology optimisation (TO) is an increasingly popular generative design method that forms an established part of the design process in various branches of industry. The density-based approach is dominant and available in several commercial software packages. TO is most often used to generate design concepts in an early stage of the design process and optimises a material distribution defined in terms of local density variables. Designs generated by density-based TO exhibit jagged and/or smeared boundaries, which forms an obstacle to their integration with existing CAD tools. How to bridge the gap between TO and CAD is a longstanding challenge. Addressing this problem by manual design adjustments or smoothing is time-consuming and affects the optimality of TO designs.

This research proposes a fully automated procedure to obtain unambiguous, accurate and optimised geometries from arbitrary 3D TO density fields. The procedure starts with a geometry extraction stage using a parametric level-set-based design description involving radial basis functions. The geometry extraction is followed by a shape optimisation stage involving local analysis refinements near the structural boundary using the Finite Cell Method (FCM). Elements located outside the structural domain are discarded to improve the computational efficiency of the shape optimisation. Well-defined bounds on basis function weights ensure that sufficient sensitivity information is available throughout the shape optimisation process. The sensitivity analysis for the shape optimisation is very similar to that of the preceding TO. This facilitates application of the proposed method post-processing method in a variety of TO

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optimisation problems with limited implementation effort. Our approach results in highly smooth and accurate optimised geometries.

To our knowledge, this is the first automated post-processing procedure for density-based TO results that applies to 3D problems and produces optimised results. Given its general formulation, it can be applied to a wide variety of problems, and allows the generation of high quality, high resolution designs at greatly reduced computational cost.

1. Introduction

Topology optimisation (TO) is an increasingly popular generative design method that forms an established part of the design process in various branches of industry. The density-based approach, introduced by Bendsøe (1989) as the SIMP method, is dominant and available in several commercial software packages. Typical density-based TO results, in combination with common filtering techniques, exhibit intermediate densities representing virtual semi-dense material, and jagged boundaries appear due to the use of a finite-element-based design discretization (e.g. Figure 1, left). Currently, TO results are subsequently redrawn or post-processed manually for further design iterations and higher fidelity analysis. Given the present efficient TO processes, this manual post-processing step increasingly becomes a bottleneck and it also compromises the optimality of the TO design. In addition, a more seamless connection between TO and existing CAD tools for e.g. design validation is desired.

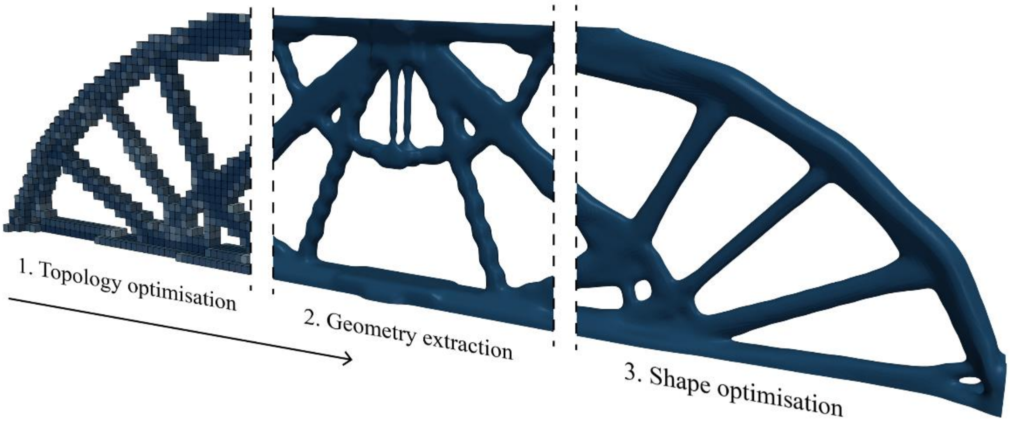

Figure 1: The three-stage structural design optimisation process illustrated on a 3D MBB beam: topology optimisation, geometry extraction and shape optimisation. This research focuses on the post-processing of TO results, i.e. Stages 2 and 3.

This research focuses on the automated post-processing of both 2D and 3D density-based TO results. The aim is to obtain a structural design optimisation process capable of generating optimised, smooth and crisp geometries with accurate, optimised performance, without any manual labour. This includes creating a mesh only once for the TO and utilizing sensitivity analysis procedures that remain essentially the same throughout the structural design optimisation process. To this end, we propose a fully integrated level-set based shape optimisation, where the initial level-set is constructed from the result of a density-based TO process. The end result will be an optimised geometry defined by a level-set function, which can be subsequently converted into other geometry representations. The novelty of our approach lies in Stage 2 and 3 of the design process shown in Figure 1: seamlessly combining geometry extraction and shape optimisation of density-based TO results.

2. Geometry extraction

In this research, a level-set function (LSF) is used as the geometry description because of its inherently smooth characteristics. Furthermore, an LSF is relatively easy to extend to 3D compared to for example spline representations and a parametric LSF makes it possible to utilize the same sensitivity analysis as used for the TO stage.

Firstly, the design represented by the optimised density distribution from the SIMP method needs to be converted into a geometry description based on an LSF. Our LSF is described by a summation of Radial Basis Functions (RBFs), similar to Luo et al. (2008), each located at the centroid x_i of an element i. Gaussian RBFs N_i will be used and are described by:

$$N_i(x) = e^{-R_i(x)^2}, \qquad \text{with} \quad R_i(x) = \|x_i - x\|_2 \,.$$

Here R_i is the radial distance from the location of the RBF. Each RBF is multiplied with a certain weight w_i to control the LSF. The LSF $\phi(\boldsymbol{x}, \boldsymbol{w})$ is the summation of the RBFs in the design domain and thus becomes:

$$\phi(\boldsymbol{x}, \boldsymbol{w}) = \sum_{i=1}^{n} e^{-R_i^2} \cdot w_i \,,$$

where n is the number of elements. Utilizing local RBF support, a sparse linear system of equations can be set up to initialize the weights of the RBFs, such that the LSF matches the densities obtained at Stage 1 in all element centroids:

$$\boldsymbol{\Phi w} = \boldsymbol{\rho}, \quad \text{where} \quad \Phi_{ij} = e^{-R_{ij}^2} \quad \text{with} \quad R_{ij} = \left\|x_i - x_j\right\|_2 ,$$

and where x_i and x_j denote respectively the centroids of elements i and j.

3. Shape optimisation

The weights of the RBFs w_i, obtained in Stage 2, can be used as design variables for a subsequent shape optimisation of the smooth geometry, similar to e.g. Luo et al. (2009). The aim of this optimisation process is to finetune the shape of the design and restore its optimality. Accurate structural and sensitivity analysis are needed in order to perform a high-fidelity shape optimisation using a level-set method (LSM) at this stage.

The structural analysis is performed using the Finite Cell Method (FCM) as introduced by Parvizian et al. (2007), see Figure 2. FCM captures both the geometry and the response of the structure more accurately than the initial low-order FE-model used in the TO process. Still FCM allows the same grid used in Stage 1 to be used while the geometry is described using an LSF.

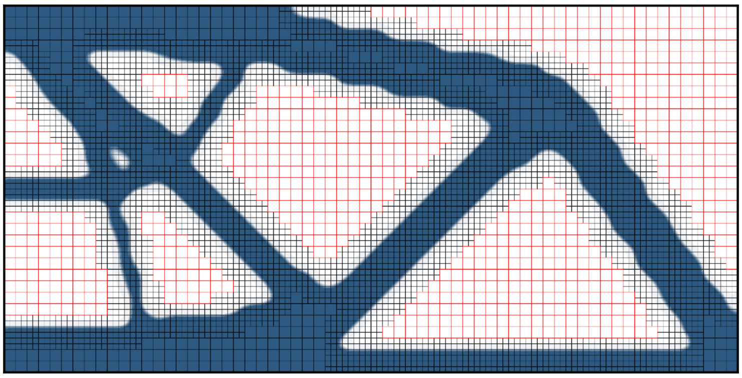

Figure 2: FCM grid on the extracted geometry of a 2D MBB beam using one level of quadtree refinement. Red squares correspond to discarded elements and black squares to integration cells.

The sensitivity analysis for Stage 3 is very similar to Stage 1. The derivative of the objective (e.g. compliance C) with respect to the design variables (RBF weights w_i) is desired. The chain rule of differentiation gives:

$$\frac{\partial C}{\partial w_i} = \frac{\partial C}{\partial \hat{\rho}_k} \frac{\partial \hat{\rho}_k}{\partial \phi_k} \frac{\partial \phi_k}{\partial w_i},$$

where a summation convention applies to index k, and $\hat{\rho}_k$ and ϕ_k denote the density and LSF-value at a particular integration point x_k, respectively. The density field $\hat{\rho}$ is linked to the LSF ϕ by:

$$\hat{\rho} = \rho_0 + (1 - \rho_0) \cdot \left(\frac{1}{1 + e^{-\kappa\phi}} \right).$$

This continuous formulation allows gradient-based optimisation, where the parameter κ is set to ensure numerical stability. New theory for this is developed in this work but omitted here for brevity. The derivative of the objective can be split into two parts: the first term is similar to that of Stage 1, while the second and third terms are problem-independent and solely related to the LSM.

4. Case studies

The MBB beam is optimised on a 64×10×32 grid, allowing a volume fraction of 10% and having a minimum compliance objective, see Figure 1. The results are visualized by creating a triangulated surface, which can be directly used to generate an STL input file for additive manufacturing or processed further by other CAD tools. A cantilever beam is also optimised but then on a 30×30×30 grid, allowing a volume fraction of 5%, see Figure 3. The loads are applied at two locations on the side plane in both vertical and transverse direction. In both case studies the proposed process generates a smooth, high-quality result from a relatively coarse 3D TO result, without any manual intervention.

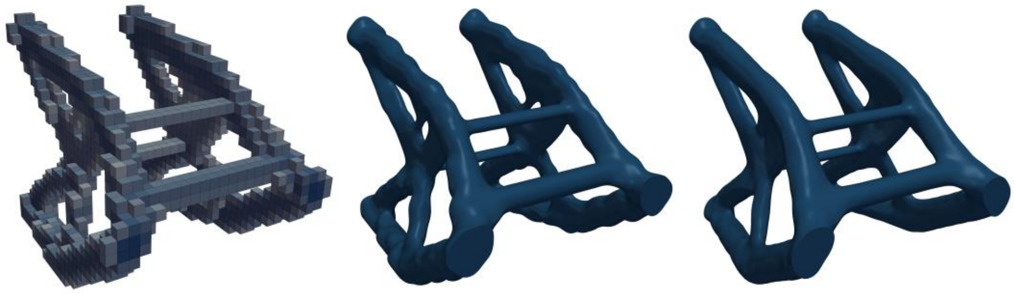

Figure 3: Three-stage structural design optimisation process (conventional TO, design conversion and shape optimisation) illustrated on a 3D cantilever beam problem.

Next to the visual evaluation, the performance of the three-stage process is also evaluated on two additional aspects: accuracy and speed. The accuracy of the final result mainly depends on the accuracy of the structural analysis, which is improved due to the use of FCM in the shape optimisation. Case studies show in our current implementation the post-processing takes more time than the low-resolution TO phase on average. However, the comparison between the computation times of Stage 1 and Stage 2+3 is not a comparison on equal grounds. To achieve similar quality (smoothness, discreteness and analysis accuracy) using conventional density-based TO alone requires extensive refinement resulting in a more than ten-fold higher computational effort.

5. Conclusion

To our knowledge, this is the first automated post-processing procedure for density-based TO results that applies to 3D problems and produces optimised results. Given its general formulation, it can be applied to a wide variety of problems, and allows the generation of high quality, high resolution designs at greatly reduced computational cost.

6. References

Bendsøe, M. P. (1989). *Optimal shape design as a material distribution problem*. Structural optimization, 1(4), 193-202.

Luo, Z., Wang, M. Y., Wang, S., & Wei, P. (2008). *A level set-based parameterization method for structural shape and topology optimization*. International Journal for Numerical Methods in Engineering, 76(1), 1-26.

Luo, Z., Tong, L., & Kang, Z. (2009). *A level set method for structural shape and topology optimization using radial basis functions*. Computers & Structures, 87(7-8), 425-434.

Parvizian, J., Düster, A., & Rank, E. (2007). *Finite cell method*. Computational Mechanics, 41(1), 121-133.