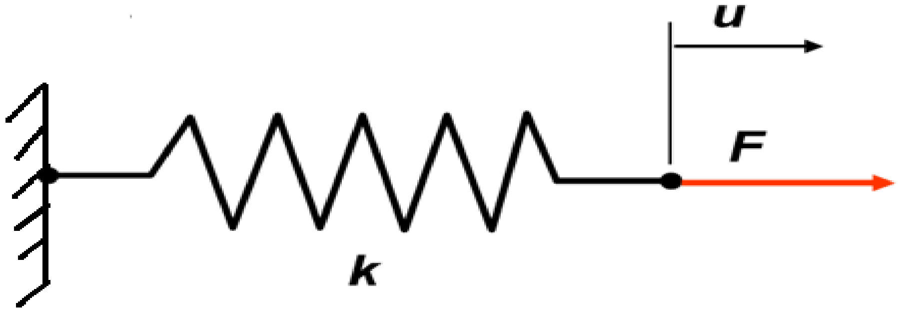
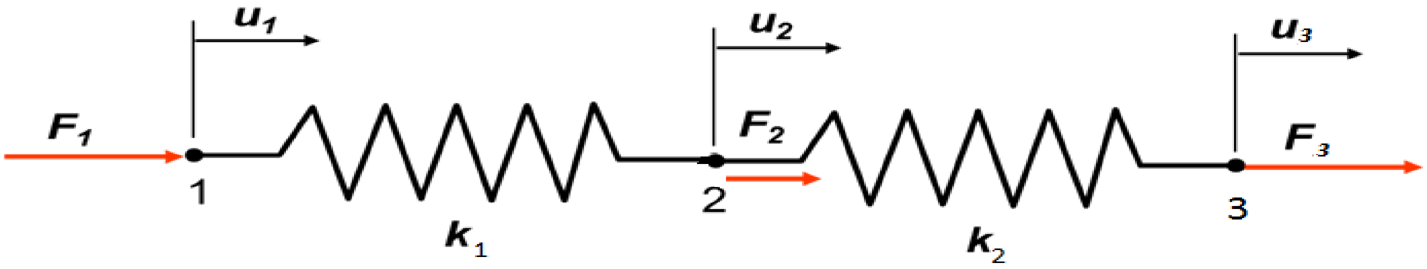
Stiffness spring



$$ku = F$$

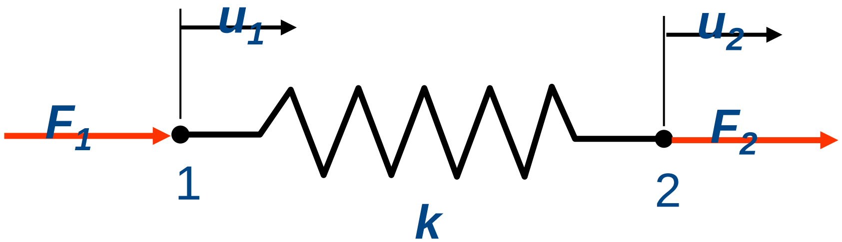
[K] = Stiffness or Property Matrix
{U} = Nodal Displacement Vector
{F} = Nodal Force Vector

Finite elements



$$[k]\{u\} = \{F\}$$

Simple Element Equation Example



Equilibrium at Node 1 $\Rightarrow \; F_1 = k(u_1 - u_2) = ku_1 - ku_2$

Equilibrium at Node 2 $\Rightarrow \; F_2 = k(-u_1 + u_2) = -ku_1 + ku_2$

or in Matrix Form

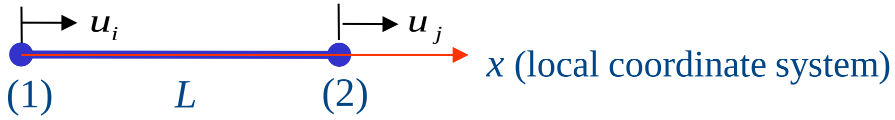
$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

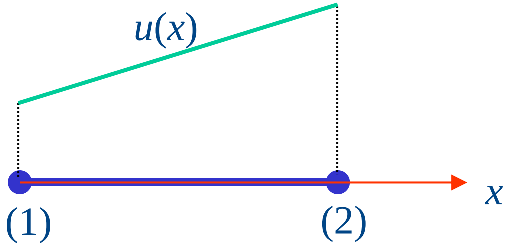
Stiffness Matrix

Nodal Force Vector

$$[K]\{u\} = \{F\}$$

Linear Approximation Scheme





Approximate Elastic Displacement

$$u = a_1 + a_2 x \quad \Rightarrow \quad \begin{aligned} u_1 &= a_1 \\ u_2 &= a_1 + a_2 L \end{aligned}$$

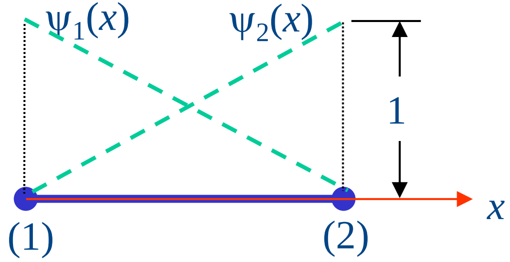
$$\Rightarrow u = u_1 + \frac{u_2 - u_1}{L} x = \left(1 - \frac{x}{L}\right) u_1 + \left(\frac{x}{L}\right) u_2$$

$$= \psi_1(x) u_1 + \psi_2(x) u_2$$

$$u = [\psi_1 \quad \psi_2] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} 1 - \frac{x}{L} & \frac{x}{L} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = [\mathbf{N}]\{u\}$$

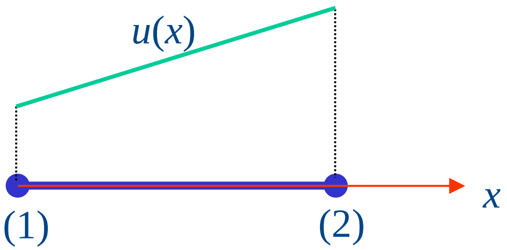
[N] =Approximation Function Matrix

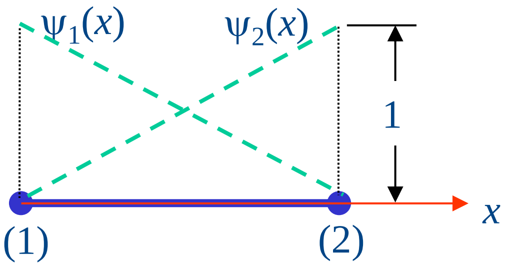
{u} = Nodal Displacement Vector



$\psi_k(x)$ – Lagrange Interpolation Functions

One-Dimensional Bar Element





$$\text{Approximation}: u = \sum_k \psi_k(x) u_k = [\mathbf{N}]\{u\}$$

$$\text{Strain}: \ e = \frac{du}{dx} = \sum_k \frac{d}{dx}\psi_k(x) u_k = \frac{d[\mathbf{N}]}{dx}\{u\} = [\mathbf{B}]\{u\}$$

$$\text{Stress-Strain Law}: \ \sigma = Ee = E[\mathbf{B}]\{u\}$$

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